

LITTLE FLOWER SCHOOL, RAPTINAGAR, GORAKHPUR
FIRST SEMESTER EXAMINATION 2021-2022

Class: XI
Mathematics

Time: 3 Hours
Max Marks: 80

SECTION A - Candidates are required to attempt all questions in this section (Internal choice has been provided in **SIX** questions)

SECTION B - Attempt all questions (internal choice has been provided in **TWO** questions)

SECTION – A

Question 1

[15x1 = 15]

In sub parts (a) to (j) choose the correct option and in sub-parts (k) to (o), answer the questions as instructed:

- a) Number of proper subsets of set $A = \{a, b, c\}$ is:
1) 6 2) 8 3) 7 4) 9
- b) If $n(A) = 3$ and $n(B) = 4$ then $n(A \times B)$ is:
1) 10 2) 25 3) 10 4) 12
- c) Given $R = \{(x, y) : y = x - 3, x, y \in Z\}$. which of the following ordered pair belong to relation:
1) (5, 2) 2) (1, 2) 3) (7, -4) 4) (-4, 1)
- d) If $f(x) = \sin x, x \in R$, then range of $f(x)$ is:
1) [-1, 1] 2) (-1, 1) 3) [0, 1] 4) [-1, 0]
- e) Number of relation on set $A = \{a, b, c\}$ is:
1) 6 2) 8 3) 7 4) None of the above
- f) The value of expression $\cos 1^\circ \cdot \cos 2^\circ \cdot \dots \cdot \cos 179^\circ$ is equal to:
1) 0 2) 2 3) 3 4) none of the above
- g) The range of $f(x) = 2 - 3x, x \in R, x > 0$ is:
1) $(-\infty, 2]$ 2) $(-\infty, 2)$ 3) $[-\infty, 2]$ 4) $[-\infty, 2)$
- h) Value of $\sin 75^\circ + \sin 15^\circ$ is:
1) $\sqrt{\frac{3}{2}}$ 2) $\frac{\sqrt{3}}{2}$ 3) $-\frac{\sqrt{3}}{2}$ 4) $-\sqrt{\frac{3}{2}}$
- i) The value of $\sin 18^\circ$
1) $\frac{\sqrt{5}+1}{4}$ 2) $\frac{\sqrt{5}-1}{4}$ 3) $\frac{\sqrt{10+2\sqrt{5}}}{4}$ 4) none of the above
- j) Consider the statement $P(n)$: $10n + 3$ is prime, then which of the following is not true:
1) $P(1)$ 2) $P(2)$ 3) $P(3)$ 4) $P(4)$
- k) If $P = \{1, -1\}$ then find $P \times P \times P$.
- l) Find the value $\cos(-870^\circ)$.
- m) If the statement $P(n) : n(n+1)(n+2)$ is divisible by 6, then what is $P(3)$.
- n) $A = \{1, 2, 3, \dots, 17\}$ and R is a relation on A defined by $R = \{(x, y) : 3x - y = 0, x, y \text{ are elements of } A\}$ then write R in roster form.
- o) Find x and y if $(4x + 3, y) = (3x + 5, -2)$.

Question 2

[2]

a) Prove that: $\sqrt{\frac{1 + \cos x}{1 - \cos x}} = \operatorname{cosec} x + \cot x$.

OR

b) If $\tan 25^\circ = x$, prove that $\frac{\tan 155^\circ - \tan 115^\circ}{1 + \tan 155^\circ \tan 115^\circ} = \frac{1+x^2}{2x}$

Question 3 [2]
If A= the set of letters in the word JAIPUR and B= the set of letters in the word JODHPUR, then verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Question 4 [2]
Prove that: $\cos 4x = 1 - 8\sin^2 x \cos^2 x$.

Question 5 [2]
a) If $f(x) = x^3 - \frac{1}{x^3}$, prove that $f(x) + f\left(\frac{1}{x}\right) = 0$

OR

b) Let $R = \{(x, y): x, y \in \mathbb{Z}, y = 2x - 4\}$, If $(a, -2)$ and $(4, b^2)$ belong to R, Find the values of a and b.

Question 6 [2]
Find the domain and range of the function $\sqrt{9 - x^2}$

Question 7 [4]
By using principle of mathematical induction prove that:

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Question 8 [4]
If ξ = the set of all digits in our decimal system, $A = \{x: x \text{ is prime}\}$, $B = \{x: x \text{ is perfect square}\}$ then prove that $(A \cup B)' = A' \cap B'$

Question 9 [4]
a) Prove that : $\sin 10^\circ \sin 50^\circ \sin 60^\circ \sin 70^\circ = \frac{\sqrt{3}}{16}$
OR

b) Prove that: $\cos A \cos 2A \cos 4A \dots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$

Question 10 [4]
a) If R is the relation on N defined by $R = \{(x, y) : y = x + \frac{12}{x}, x, y \in \mathbb{N}\}$, then find R in roster form and find its domain and range.

OR

b) If $f(x) = y = \frac{ax-b}{cx-a}$, then prove that $f(y) = x$.

Question 11 [6]
a) Find the general solution : $2 \sin^2 x + \sqrt{3} \cos x + 1 = 0$
OR

b) Find the general solution: $\cos 3x + \cos x - \cos 2x = 0$

Question 12 [6]
By using principle of mathematical induction, show that $3^{2n+2} - 8n - 9$ is divisible by 64.

Question 13**[6]**

a) Prove that $\frac{a^2 \sin(B-C)}{\sin B + \sin C} + \frac{b^2 \sin(C-A)}{\sin C + \sin A} + \frac{c^2 \sin(A-B)}{\sin A + \sin B} = 0$

OR

b) If $\tan \frac{x}{2} = \frac{m}{n}$ prove that $m \sin x + n \cos x = n$

Question 14**[6]**

In an university, out of 100 students, 15 offered Mathematics only, 12 offered statistics only, 8 offered only physics; 40 offered Mathematics and Physics, 20 offered physics and statistics; 10 offered Mathematics and statistics. 65 offered physics. By drawing a Venn diagram, find the number of student who:

- Offered Mathematics.
- Offered Statistics.
- Did not offer any of the above three subjects.

SECTION – B**Question 15****[5x1=5]**

In sub parts (a) and (b) choose the correct option and in sub – parts (c) to (e), answer the questions as instructed:

- Which of the following is not correct?
(1) $N \subset R$ (2) $N \subset Q$ (3) $Q \subset R$ (4) $N \subset T$
- Let $n(A) = m$ and $n(B) = n$, then the number of non empty relations from A to B is:
(1) m^m (2) $n^m - 1$ (3) $2^{mn} - 1$ (4) 2^{mn}
- Evaluate $\cos 15^\circ - \sin 15^\circ$.
- Find the domain of $f(x) = \frac{3x}{28-x}$.
- Prove that $(\sin x - \cos x)^2 = 1 - \sin 2x$

Question 16**[4]**

a) Solve it: $[3x - 4] = 5$

OR

b) If f and g are two real function defined by $f(x) = 2x + 1$ and $g(x) = 4x - 7$, then for what real value of x , $f(x) = g(x)$

Question 17**[2]**

a) Find the degree measure of 6 radian

OR

b) Express the angle $-37^\circ 30'$ in radian measure.

Question 18**[4]**

Draw the graph of $2\sin 2x$.