Class: IX   **Time: 2.5 Hours**
Mathematics   **Max Marks: 80**

(Attempt all questions from Section A and any four questions from Section B. All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks)

SECTION A (40 Marks)
(Attempt all questions from this Section.)

Question 1   **(3+3+4)**

a) Factorise it : $x^4 + 5x^2 + 9$

b) Simplify it:
$$\left(x^{\frac{1}{3}} - x^{-\frac{1}{3}} \right)\left(x^{\frac{2}{3}} + 1 + x^{-\frac{2}{3}} \right)$$

c) If $\dfrac{x^2 + 1}{x} = 4$, find the value of $2x^3 + \dfrac{2}{x^3}$

Question 2   **(3+3+4)**

a) Factorise: $x^3 + 3x^2 y + 3xy^2 + 2y^3$

b) Solve the following system of simultaneous linear equations
   $9 - (x - 4) = y + 7$, $2(x + y) = 4 - 3y$

c) If $a + b = 10$ and $a^2 + b^2 = 58$, find the value of $a - b$ and $a^2 - b^2$

Question 3   **(3+3+4)**

a) If $2^x = 3^y = 12^z$, prove that $x = \dfrac{2yz}{y - z}$

b) Factorise it : $64x^4 - 81y^4$

c) Solve these equation by elimination method**:**
$$\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$$
$$\frac{2}{x} - \frac{1}{y} = 1$$

Question 4   **(3+3+4)**

a) Simplify it:
$$\left(\frac{x^{a+b}}{x^c} \right)^{a-b} \left(\frac{x^{b+c}}{x^a} \right)^{b-c} \left(\frac{x^{c+a}}{x^b} \right)^{c-a}$$

b) If $x + y = 8$ and $xy = 3\dfrac{3}{4}$ find the value of $5 (x^2 + y^2) + 4 (x - y)$

c) Solve by cross multiplication method
$$x - 3y - 7 = 0$$
$$3x - 3y = 15$$

Attempt **any four** questions from this Section.

Question 5 (3+3+4)

a) If $2(x^2+1)=5x$ find; $(i)\, x-\dfrac{1}{x}$ $(ii)\, x^3-\dfrac{1}{x^3}$

b) Factorise it : $a^4 - b^4 + 2b^2 - 1$

c) The sides of an equilateral triangle are $2x-3y+1, x+y-1 \;and\; 3x-y-9$. Find the length of each side.

Question 6 (3+3+4)

a) Given that $1176 = 2^p.3^q.7^r$, find
   (i) the numerical value of p, q and r   (ii) find the value of $2^p.3^q.7^{-r}$ as a fraction.

b) If $a + 2b = 5$, prove that $a^3 + 8b^3 +30ab = 125$.

c) Find the expansion of (i) $\left(\dfrac{2}{3}x-\dfrac{3}{2x}-1\right)^2$ (ii) $\left(2x-\dfrac{1}{3y}\right)^3$

Question 7 (3+3+4)

a) If $a^2-3a-1=0$, find the value of $a^2+\dfrac{1}{a^2}$

b) Factorise it $(4 - x)^2 - 2x$

c) Solve: $\dfrac{20}{x+1}+\dfrac{4}{y-1}=5, \dfrac{10}{x+1}-\dfrac{4}{y-1}=1$

Question 8 (3+3+4)

a) If $a+b-c=5$ and $a^2+b^2+c^2=29$ find the value of $ab-bc-ca$

b) If x = 5 - 2√6, find the value of $\sqrt{x}+\dfrac{1}{\sqrt{x}}$

c) A number of two digits exceeds four times the sum of its digits by 6 and it is increased by 9 on reversing the digits . find the number.

Question 9 (3+3+4)

a) Factorise: (i) x^4+4 (ii) x^4+x^2+1

b) Show that $\dfrac{a^{-1}}{a^{-1}+b^{-1}}+\dfrac{a^{-1}}{a^{-1}-b^{-1}}=\dfrac{2b^2}{b^2-a^2}$

c) A boat takes 2 hours to go 40 km down stream and it returns in 4 hours. Find the speed of boat in still water and the speed of stream.

Question 10 (3+3+4)

a) Factorise: $x^4 -14x^2y^2 - 51y^4$

b) Solve: $3x+14y=5xy \;and\; 21y-x=2xy$

c) Solve the equation: $3(2^x +1) - 2^{x+2} + 5 = 0$.

Question 11 (5+5)

a) Solve: $4x+\dfrac{x-y}{8}=17 \,and\, 2y+x-\dfrac{5y+2}{3}=2$

b) If $x - y = 7$ and $x^2 + y^2 = 85$, then find the value of $x^3 -y^3$.